How to cite this article:

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Title of article: "Influence of modification of an asymmetric outline on teeth deformation in the spur gears"

Mechanik, No. 7 (2019)DOI: <https://doi.org/10.17814/mechanik.2019.7.62>

Influence of modification of an asymmetric outline on teeth deformation in the spur gears

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The aim of the article was to determine the value of deformation of straight teeth with an asymmetric outline with variation of the pressure angle and minimization of tooth undercutting. The geometry of the gear was developed as a parametric model in the NX 10 system. Simulation tests of tooth deflection were performed using the Pre/Pos module available in the NX10 program.

KEYWORDS: gears, asymmetric tooth, FEM, 5-axis milling

To reduce the deformation of the elastic teeth of gears under load, their outline is often modified. One of the modification methods is to introduce an asymmetric tooth outline. Previous FEM studies focused mainly on the analysis of deformation and strength of asymmetric teeth while modifying the transition curve at the base of the tooth [1]. A commonly used method of gear tooth modification is to change the pressure angle α_1 and α_2 on the tool, which is the Maag gear (fig. 1). Parameters of this modification are presented in tab. I [1, 2].

Fig. 1. Geometry of Maag gear

TABLE I. Sizes defining a gear tool

Designation	Pattern	Description
α	–	pressure angle
h	$h = h_g + h_a$	tooth height
h_a	$h_a = (y + x) \cdot m$	addendum
h_f	$h_f = (y - x) \cdot m + c$	dedendum
m	–	module
p	$p = \pi \cdot m$	circular pitch
s	$s = p/2$	tooth thickness
l_w	$l_w = c \cdot m$	top clearance

where: c – vertex clearance factor, y – tooth height factor, x – tooth correction factor

Fig. 2. Tooth geometry with undercut marked

This method, however, has a major disadvantage, namely: as a result of the rolling away of the rack gear at small pressure angles, i.e. $\alpha_2 < 20^\circ$, an undercut of the tooth is created, weakening its base (fig. 2). Whether an undercut occurs also depends on the limit number of teeth, which for a gear tool is determined by the relationship [2, 3]:

$$z_g = y \frac{2}{\sin^2 \alpha_i} \quad (1)$$

where: y - tooth height factor, α_i - pressure angle ($i = 1, 2$).

Therefore, new methods of asymmetric tooth outline modification are being sought. One of them is the machining of teeth on five-axis machines, which allows for obtaining the asymmetric modification of the tooth profile as a result of a change in the tilt angle of the cutter axis β (fig. 3). Machining is carried out by tangentially positioning the cutter to the side surface of the tooth. The main feed movement takes place along the tooth line, and the positioned tool axis is implemented by changing the angle β . This angle changes depending on the position of the tool on the tooth contour line. The range of angle β variation is limited by the diameter of the vertices d_a and the diameter of the feet d_f (tab. II). As a result of the β angle control, the required tooth side curvature and the specified angle of the action line can be obtained.

The change of angle β depending on the diameter d_i , on which this angle is determined, is described by the equation:

$$\beta = \arccos\left(\frac{\cos \alpha \cdot d}{d_i}\right) + \tan\left(\arccos\left(\frac{\cos \alpha \cdot d}{d_i}\right)\right) \cdot \frac{180}{\pi - \arccos\left(\frac{\cos \alpha \cdot d}{d_i}\right)} \quad (2)$$

This relationship makes it possible to determine the β value for each gear diameter d_i between the limiting values d_a and d_f .

TABLE II. Characteristic diameters in gear

Designation	Pattern	Description
d	$d = m \cdot z$	pitch diameter
d_a	$d_a = d + 2 \cdot h_a$	tip diameter
d_f	$d_f = d - 2 \cdot h_f$	base diameter
d_b	$d_b = \cos \alpha \cdot d$	root diameter

The course of β angle change depending on the diameter, on which the tool works, and the change of the α_1 pressure angle are shown in fig. 3. It should be noted that the relationship (2) applies in the case of the α_2 pressure angle for the opposite side of the tooth.

Fig. 3. Kinematics of five-axis machining for the left and right side of the tooth (a) and positioning of the tool for the left and right flank of the tooth (b)

Taking into account the cited dependencies, a parametric CAD model of the gear was developed with the module $m_n = 5$ and the number of teeth $z = 18$, which allows for obtaining asymmetry of processed teeth with different pressure angles - both for the active side and the passive side of the tooth [4].

FEM analysis

Tooth deformation tests for geometry with limited undercutting were performed in the NX Pre/Pos module. As variable parameters in the first case, the pressure angle on the active flank of the tooth α_1 was adopted with a constant angle on the passive flank α_2 . In the second case, the reverse was considered. FEM analysis was performed on a single tooth, for which a 2 mm Tetrahedral 3D mesh model was adopted, with local density on the force interaction surface and on the transition curve, at the level of 0.5 mm (fig. 4).

Fig. 4. Method of applying the force to the examined tooth

The test consisted in applying a concentrated force $P = 1000 \text{ N}$ to the active surface of the tooth, acting along the action line, and determining the value of the deflection f the tooth at its tip. Steel with Young's modulus $E = 206 \text{ GPa}$, Poisson's ratio $\nu = 0.3$ and Kirchhoff modulus $G = 80 \text{ GPa}$ [1, 5, 6], was adopted as the gear material.

In the case of an asymmetric tooth profile, there is a non-uniformity of force acting on the action line. This is due to the number of pairs of teeth working together at any given time. In this case, the greatest forces occur in the middle area of the tooth outline. Force P was applied in the upper part of the area, in which only one pair of teeth cooperate, so that it is possible to determine the maximum deformation of the tooth [5]. Eleven tests were carried out for the adopted parameters - for each case (fig. 5).

Fig. 5. Graph of the dependence of the tool's tilt angle β on the position taken

In the first case, angle α_1 was increased by 2° in the range from 20° to 40° , with a constant value of angle α_2 of 20° . In the second case, the α_2 pressure angle on the tooth's passive surface was changed in the range from 10° to 20° by 1° , while the angle α_1 was 40° .

The limit values - $\alpha_1 = 40^\circ$, $\alpha_2 = 20^\circ$ - were selected in such a way as to prevent shearing the tooth tip. Changes in tooth deformation at subsequent values of angle α_1 are shown in fig. 6.

Fig. 6. Changes in tooth deformation at subsequent values of angle α_1

By introducing different preasure angles α_1 and α_2 , the cross-sectional area of the tooth at its base was changed. In the first case, for $\alpha_1 = 40^\circ$, there is the largest cross-sectional area at the base of the tooth and the smallest deformation f .

Analysis of simulation results of tooth deformation change

Based on the simulation results, charts were developed showing the value of tooth deformation f with a variable angle on the active flank α_1 (fig. 7) and on the passive flank of tooth α_2 (fig. 8).

Fig. 7. Diagram of the dependence of deformation f on the preasure angle α_1

Fig. 8. Diagram of the dependence of deformation f on the preasure angle α_2

The graphs show that as the value of angle α_1 increased, the value of the tooth's f deformation at its tip decreased twice. When the angle α_2 changes in the range of 10° to 20° , deformation shows a downward trend, but with a much milder character. The smallest deformation value was observed for the pair of angles α_1 and α_2 equal to 40° and 10° , respectively, and the highest for the pair of 20° and 20° .

With constant material and geometrical parameters, the change for the preasure angle α_1 , defined for the active flank of the tooth, was described by the formula (3) as an exponential function, and for the angle α_2 , describing the passive flank of the tooth, it shows a polynomial form expressed by equation (4):

$$f = 0,0043 \cdot e^{-0,033\alpha_1} \quad (3)$$

$$f = -10^{-6} \cdot \alpha_2^2 + 10^{-5} \cdot \alpha_2 + 0,0014 \quad (4)$$

Conclusions

Possibility of modifying the tooth profile in the kinematics of a five-axis machine tool by tangentially guiding the tool to the machined surface and using the tilting angle of the tool axis β was confirmed, which allows for obtaining the desired values of the α angle on both flank of the tooth.

Modification of the gear tooth profile has a significant impact on reducing tooth deformation and, consequently, on increasing the strength of the entire transmission. This involves increasing the ranges of transmitted power, as well as slimming the entire structure.

The analysis showed a much smaller impact of angle α_2 on tooth tip deformation, however, the change in the value of this angle is dictated by geometric relationships and is achieved by obtaining the largest angle α_1 with the elimination of tooth tip shearing. The case studied, referring to a straight-tooth wheel, indicates that this type of modification should be used on heavily loaded wheels, also with helical teeth, as well as on bevel gears.

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